EXPERIENTIA

Vol. XII - Fasc. 6 Pag. 205–244 15. VI. 1956

Brèves communications - Kurze Mitteilungen
Brevi comunicazioni - Brief Reports

The Wilcoxon and Related Tests of Significance

VAN DER VAART[1] has given exact expressions for the probability of obtaining a result in the Wilcoxon two-sample test deviating by a given amount or more from that expected, when the two samples are drawn from the same population. In his terminology, if n_1 and n_2 are the numbers in the two samples, and U_{12} is the number of pairs of individuals, in which the value observed for the member of the first sample exceeds the value observed for the second, he finds the exact probability that $U_{12} \leqslant u$, where u is the observed value. To do so, it is necessary to evaluate two determinants of degree u. If u is fairly large, for example 23 in one of his examples, "the calculations" he writes "may turn out to be somewhat cumbersome".

HALDANE and SMITH[2] have solved this problem approximately. But as their work was concerned with a question of human genetics, it seems to have escaped the notice of some statisticians. In VAN DER VAART's terminology, and choosing $n_2 \leqslant n_1$, their equations (10) and (11) become:

$$X = \tfrac{1}{2}n_2 + (u + \tfrac{1}{2} - \tfrac{1}{2}n_1 n_2)\,[n_1(n_1+n_2+1)]^{-\frac{1}{2}}$$

and

$$P\,(U_{12} \leqslant u) = \frac{X^{n_2}}{n_2\,!\,0!} - \frac{(X-1)^{n_2}}{(n_2-1)\,!\,1!} + \frac{(X-2)^{n_2}}{(n_2-2)\,!\,2!} - \cdots\cdots + \frac{(-1)^r\,(X-r)^{n_2}}{(n_2-r)\,!\,r!},$$

where r is the greatest integer not exceeding X. This expression is not very accurate when n_2 is less than about 4, nor for very small values of P. But, as the following examples show, it is good enough for practical purposes over the range of values of P which are usually taken as critical in tests of significance. VAN DER VAART gives two numerical examples.

For $n_1 = 49$, $n_2 = 2$, $u = 7$, he finds

$$P\,(U_{12} \leqslant 7) = 0{\cdot}015686\,.$$

Our formula gives $X = 0{\cdot}17784$, $P = 0{\cdot}01581$.

For $n_1 = 21$, $n_2 = 5$, $u = 23$, he finds

$$P\,(U_{12} \leqslant 23) = 0{\cdot}028717\,.$$

Our formula gives $X = 1{\cdot}2821$, $P = 0{\cdot}02863$.

In the first case only one term of the series was required, in the second two. The calculation takes about 5 min. We also gave methods for combining the results of several pairs of samples, for dealing with "ties", or equality between members of the two samples, and so on. VAN DER VAART's method, being exact, is perhaps preferable to ours for values of u up to 4 or 5, but I think that at higher values the simplicity of our method makes up for its slight inaccuracy.

J. B. S. HALDANE

Department of Biometry, University College, London, February 23, 1956.

Résumé

Une formule assez simple permet le calcul rapide des probabilités dans le test de WILCOXON et d'autres méthodes semblables. Sans fournir des résultats exacts, elle donne, dans les exemplaires choisis, des chiffres qui se rapprochent de la formule exacte de VAN DER VAART, avec des erreurs inférieures à 1%.

[1] H. R. VAN DER VAART, Exper. *12*, 14 (1956).

[2] J. B. S. HALDANE and C. A. B. SMITH, Ann. Eugen. *14*, 117 (1948).

The Structure of Isothebaine

The alkaloid isothebaine, isolated by GADAMER[1] from the roots of *Papaver orientale* during the period of withering and rest of the plant, was examined in detail by KLEE[2], who degraded the methyl ether through a trimethoxyvinylphenanthrene to a trimethoxyphenanthrene carboxylic acid, which on decarboxylation afforded 3:4:5-trimethoxyphenanthrene (I); the work was repeated and the identity of the final product confirmed by SCHLITTLER and MÜLLER[3]. In view of the agreement of these independent workers the contention of KISELEV and KONOVALOVA[4] that the end-product of the degradation is not identical with 3:4:5-trimethoxyphenanthrene may be discounted. On the basis of this degradation isothebaine methyl ether has been hitherto allotted the structure (II, R = Me)[5], but attempts to

[1] J. GADAMER, Arch. Pharm. *249*, 41 (1911).

[2] W. KLEE, Arch. Pharm. *252*, 211 (1914).

[3] E. SCHLITTLER and J. MÜLLER, Helv. chim. Acta *31*, 1119 (1948).

[4] V. V. KISELEV and R. A. KONOVALOVA, J. gen. Chem. (USSR.) *19*, 148 (1949).

[5] W. KLEE, Arch. Pharm. *252*, 211 (1914). – E. SCHLITTLER and J. MÜLLER, Helv. chim. Acta *31*, 1119 (1948).